Hamilton-connected Indices of a Class of Graphs Obtained from the Petersen Graphs

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Abstract: In this paper, we mainly considered the Hamilton-connected indices of the Petersen graph and the graphs obtained by replacing each vertices of the Petersen graph with an *n*-cycle. Hamilton-connected index is the minimum integer *m* of the *m*-time iterated line graph $L^m(G)$ of Petersen graph classes such that $L^m(G)$ is Hamilton-connected. A graph *G* is Hamilton-connected if any two vertices of *G* are connected by a Hamilton path. We show that the Hamilton-connected indices of the Petersen graphs is 1, and the Hamilton-connected indicies of graphs obtained by replacing each vertex of the Petersen graph with an *n*-cycle (3≤*n*≤6) is 2.

Keywords: Petersen graph; Iterated line graph; Hamilton-connected index

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1. Introduction

In this paper we consider finite undirected simple graphs and follow the notation and terminology of [1].

Let *G* be a graph with vertex set $V(G)$ and edge set $E(G)$ The line graph $L(G)$ of a graph *G* is the graph with vertex set $E(G)$, in which two vertices are adjacent, if and only if the corresponding edges have a common end vertex in *G*. For $m \geq 1$, the *m*-time iterated line graph $L^m(G)$ is defined recursively by $L^0(G)=G$ $L^1(G)=L(G)$ and $L(L^m(G)) = L(L^{m-1}(G))$. A graph is *Hamilton-connected* if for any two vertices $u,v \in V(G)$ there exists a *(u,v)-path* containing all vertices of *G*. The *Hamilton- connected index* is the smallest integer *m* such that $L^m(G)$ is Hamilton-connected, denoted by *hc(G)*. A path passing through all the vertices of a graph is called a *Hamilton path*. The path, cycle and complete graph with *n* vertices are denoted P_n, C_n and K_n, respectively. Defined $V_i(G)=\{v \in V(G): d_G(v)=i\}$ .The diameter of *G* is $diam(G)=max_{v\in V(G)}\{max\{d(v,w)/w\in V(G)\}\}$. we use $\kappa(G)$ and $\kappa'(G)$ to denote the connectivity and the edge-connectivity of *G*. For an integer $k > 0$, a graph *G* is *essentially k -edge -connected* if *G* does not have an essential edge cut *X* with $|X| < k$. The Petersen Graph is the simple graph whose vertices are the 2-element subsets of a 5-element set and whose edges are the pairs of disjoint 2-element subsets.

A lane in *G* is a nontrivial whose ends are not in $V_2(G)$ and whose internal vertices, if any, have degree 2 in *G*. $\mathrm{dist}(e_i, e_j)$ is the length of the shortest path from a certain end point of to certain end point of e_j. A *poly-vertex* is one having degree at least 3. A *poly-path* is a path joining two poly-vertices of *G*, whose internal vertices, if any, have degree 2. A *cyclic-poly-path* is a path such that the ends of it are poly vertices and all internal vertices are on a cycle and of degree 2. An *end-vertex* is a leaf or one vertex having degree 1. An end-path is one joining a poly-vertex with an end-vertex such that whose internal vertices, if any, have degree 2.

A subgraph *H* of a graph *G* is dominating, if $G-V(H)$ is edgeless. Let $v_0, v_k \in V(G)$, a (v_0, v_k) -trail of *G* is a vertex-

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edge alternating sequence $v_0e_1v_1e_2\ldots e_kv_k$, such that all the e_i are distinct and for each i=1, 2,...,k, e_i joins v_{i-1} with v_i. With the notation above, the (v_0,v_k) -trail is also called an (e_1,e_k) -trail, and the vertices $v_1,v_2,\ldots,v_{k-1}$ are called internal vertices of the trail. When $(v_0=v_k)$ the (v_0,v_k)-trail is called a (v_0,v_k) closed trail. A *dominating* (e_i,e_k) *- trail T* of *G* is an (e_i,e_k) - trail such that every edge of *G* is incident with an internal vertex of *T*. A *spanning* (e_1,e_k) *-trail* of *G* is an (e_1,e_k) -trial such that $V(T)=V(G)$ A dominating closed trail (abbreviated DCT) of *G* is a closed trail (or, equivalently, an eulerian subgraph) *T* in *G* such that every edge of *G* has at least one vertex on *T*. There is a close relationship between dominating closed trails in graph *G* and Hamilton cycles in *L(G)*, as follows:

Theorem 1. [2] Let *G* be a graph with $|E(G)| \geq 3$.Then *L(G)* is hamiltonian if and only if *G* has a dominating closed trail.

In 1983, Clark and Wormald in [3] generalized the definition regarding Hamiltonian indices and introduced the notion of Hamilton-connected index. In 2009, Chen et.al. in [4] studied the Hamilton-connected index problem and obtained a necessary and suffcient condition, stated as follows:

Theorem 2. [4] Let *G* be a graph with $|E(G)| \geq 3$.Then *L(G)* is Hamilton-connected if and only if for any pair of edges $e_1, e_2 \in E(G)$, *G* has a dominating (e_1, e_2)-trail.

It is obviously that the following result:

Corollary 3. If *G* is Hamilton-connected, then *L(G)* is also Hamilton-connected.

In [4], Chen, et.al. established certain relationships between the Hamilton-connected index and both the minimum and maximum degrees of a graph. Additionally, they presented some relations between the Hamilton-connected index and the connectivity and diameter of *G*, and results as follows:

Theorem 4. [4] Let *G* be a connected graph with minimum degree at least 3. Then $hc(G) \leq 3$.

Theorem 5. [4] Let *G* be a graph which is neither a path nor a cycle. Then $\kappa^3(G) \leq hc(G) \leq \kappa^3(G)+2$, where $\kappa^3(G)=\min\{m \mid L^m(G)\ is\ 3-connected\}$.

Theorem 6. [4] Let *G* be a connected graph that is neither a path nor cycle. If the length of a longest lane is *k* with $k \geq diam(G)+1$, then $hc(G)=k-1$.

In 2014, Sabir Eminjan and Vumar Elkin in [5] introduced the relations between the Hamilton-connected index and the Hamiltonian index of trees. After that, they determined the upper and lower bounds of the Hamilton-connected index of unicyclic graphs. They also presented the trees and unicyclic graphs *G* with the property that $hc(G)=h(G)+1$, the results as follows:

Theorem 7. [5] If *T* is a tree with $|V(T)| \geq 5$ which is not a path, then $h(T) \leq hc(T) \leq h(T)+1$, where $h(T)=max\{\{l(P)+1\}, \{l(Q)\}\}$ over all poly-path *P* and end-paths *Q* of *T*, and *l*(.) is the length of a path.

Theorem 8. [5] Let *G* be a unicyclic graph with $|V(G)| \geq 4$ that is not a cycle. Then, $k \leq hc(G) \leq max\{k+1, k'+1\}$, where $k = max\{\{l(P)+1\}, \{l(Q)\}\}$ overall poly-paths *P* and end-paths *Q* of *G*, *l*(.) is length of a path, and k' is the length of a longest cyclic-poly-path in *G*.

Theorem 9. [5] If *G* is a connected graph containing cyclic and acyclic blocks such that each cyclic block is hamiltonian, then $k \leq hc(G) \leq max\{k+1, k'+1\}$, where $k= max\{\{l(P)+1\}, \{l(Q)\}\}$ over all poly-paths *P* and end-paths *Q* of *G*, *l(.)* is length of a path, and k', is the length of a longest cyclic-poly-path in *G*.

As we well know, the determination of a Hamilton-connected graph is an NP-hard problem, there are few results regarding the Hamilton-connected index of graph, we looked up some results between Hamilton-connected graph and the line graph, as follows:

Theorem 10. [6] If *G* is a 4-edge-connected graph, then the line graph *L(G)* is Hamilton-connected.

Theorem 11. [7] Every 7-connected line graph is Hamilton-connected.

Theorem 12. [8] (I) Every 4-connected line graph of a claw free graph is Hamilton-connected.

(II) Every 4-connected hourglass free line graph is Hamilton-connected.

Theorem 13. [8] Let *G* be a graph such that *L(G)* is 4-connected and every vertex of degree 3 in a triangle of *G*, then *L(G)* is Hamilton-connected.

Theorem 14. [9] Every 3-connected, essentially 11-connected line graph is Hamilton- connected.

In 2023, Lv and Zhao in [10] provided some results on the Hamiltonian indices of three classes of graphs obtained from Petersen graph, one of results as follows:

Theorem 15 [10] (I) Let *G* be a Petersen graph, then *h (G)*=1.

(II) Let *G* be the graph obtained by replacing every vertex of Petersen graph with a *n*-cycle, then *h (G)*=2.

(III) Let *G* be the graph obtained by adding n pendant edges to each vertex of Petersen graph, then *h(G)*=2.

2. Our Main Results

Motivated by these studies, we consider the Hamilton-connected indices of the Petersen graph and the graphs obtained by replacing each vertices of the Petersen graph with an *n*-cycle, and obtain the following results:

Theorem 16. Let *G* be a Petersen graph, then *hc (G)*=1.

Theorem 17. Let *G* be the graph obtained by replacing every vertex of Petersen graph with a *n*-cycle (3≤*n*≤6), then *hc (G)*=2.

Theorem 18. Let *G* be the graph obtained by replacing every vertex of Petersen graph with a *n*-cycle(*n* ≥ 7), then *hc (G)*≥2.

3. Proof of Main Results

Proof of theorem 16 First of all, the Petersen graph *G* is a 3-connected graph. According to Theorem 5, we can obtain 0 ≤ *hc (G)* ≤ 2. As we all know, the Petersen graph is not a Hamilton-connected graph, then *hc (G)* ≠ 0, theorefore 1 ≤ *hc (G)* ≤ 2. Next, we prove *hc(G)*=1by Theorem 2, we need to find a dominating (e_i, e_j) -trail for any pair of edges in the Petersen graph. Since the diameter of the Petersen graph is 2, we can be certain that the dominating (e_i, e_j) -trail has 3 cases in *G*, as follows:

For convenience, we mark the vertices and edges of Petersen graph as figure 1, Let $u_n, v_n (n = 0,1,2,3,4)$ be respectively vertices of outer cycle C_5 and inner cycle C_5, $e_n, e_n^{'}, f_n (n = 0,1,2,3,4)$ be respectively edges of outer cycle C_5 and inner cycle C_5 and the edges connecting the outer cycle C_5 and the inner cycle C_5.

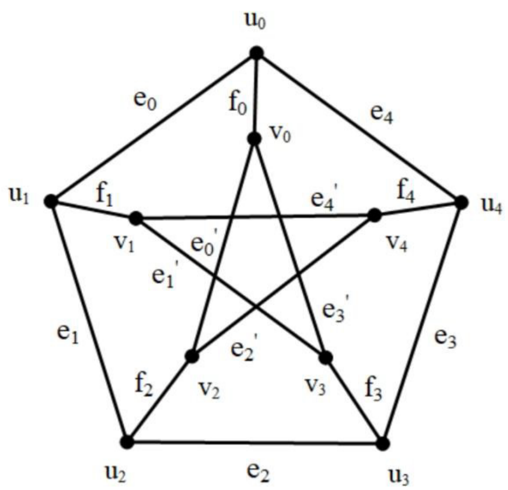

Figure 1: Petersen Graph

Case 1. When $\text{dist}(e_i, e_j) = 0$, it means we have found two adjacent edges in the *G*, Since the Petersen graph is a Symmetric figure, some isomorphic structures will not be elaborated further in this part of the text.

Subcase 1.1 e_i and e_j are the edges of outer C_5 of the Petersen graph, there exist two situations where two sides with a distance of 0 which are adjacent to each other. There are 5 pairs of adjacent (e_i, e_j) edges, which can be separated into two types: (e_4, e_0) and $(e_n, e_{n+1})(n=0,1,2,3)$ For any (e_i, e_j) , we can find a dominating (e_i, e_j) trail in figure 1. Taking (e_0, e_1) for instance, we provide one of the edge-trail sequence as follows: $e_0 u_0 e_4 u_4 e_3 u_3 e_2 u_2 f_2 v_2 e_0' v_0 e_3' v_3 e_1' v_1 f_1 u_1 e_1$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 1.2 e_i and e_j are also two situations among the adjacent edges between the outer C_5 of the Petersen graph and the internal edges of the connected C_5. There are 10 different adjacent (e_i, e_j) edge pairs, which can be separated into three types: (e_n, f_n) , $(e_n, f_{n+1})(n=0,1,2,3)$, and (e_4, f_0) For any (e_i, e_j) , we can find a dominating (e_i, e_j) -trail in figure 1. Taking (e_0, f_0) for instance, we provide one of the edge-trail sequences: $e_0 u_1 e_1 u_2 e_2 u_3 e_3 u_4 f_4 v_4 e_4' v_1 e_1' v_3 e_3' v_0 f_0$ Similarly, we C_5 can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 1.3 e_i is the edge connecting the outer and the inner C_5 ,and e_j is the edge on the inner C_5 of the Petersen graph. There are 10 different adjacent (e_i, e_j) edge pairs, which can be separated into three types: (f_n, e_n') , $(f_n, e_{n+3}')(n=0,1)$ and $(f_n, e_{n-2}')(n = 2,3,4)$ For any (e_i, e_j) , we can find a dominating (e_i, e_j) -trail in figure 1. Taking (f_0, e_0') as an example, we provide one of the edge-trail sequence: $f_0 u_0 e_0 u_1 e_1 u_2 e_2 u_3 f_3 v_3 e_1' v_1 e_4' v_4 e_2' v_2 e_0'$ Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 1.4 Both e_i and e_j are edges on the inner C_5 of the Petersen graph. There are 5 different adjacent (e_i, e_j) edge pairs, which can be separated into three types: $(e_n', e_{n+3}')(n=0,1)$, $(e_n', e_{n+2}')(n=2,3,4)$. For any (e_i, e_j) , we can find a dominating (e_i, e_j) -trail in figure 1. Taking (e_0', e_3') as an example, we only provide one of the edge-trail sequences: $e_0' v_2 e_2' v_4 e_4' v_1 e_1' v_3 f_3 u_3 e_2 u_2 e_1 u_1 e_0 u_0 f_0 v_0 e_3'$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Case 2. $\text{dist}(e_i, e_j) = 1$ in the Petersen graph.

Subcase 2.1 e_i and e_j are non-adjacent edges on the outer C_5 of the Petersen graph with distance of 1. There are 5 different adjacent (e_i, e_j) edge pairs, which can be separated into two types: $(e_n, e_{n+2})(n=0,1,2)$ and $(e_n, e_{n-3})(n=3,4)$. For any (e_i, e_j) , we can find a dominating (e_i, e_j) -trail in figure 1. Taking (e_0, e_2) as an example, we list the dominating edge trail sequence: $e_0 u_1 f_1 v_1 e_1' v_3 e_3' v_0 e_0' v_2 e_2' v_4 f_4 u_4 e_3 u_3 e_2$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 2.2 e_i is the edge on the outer C_5 of the Petersen graph, and e_j is the edge on the inner C_5. Except for five edge pairs: $(e_n, e_{n+2}')(n=0,1,2)$, $(e_n, e_{n-3}')(n=3,4)$, there are 20 different (e_i, e_j) edge pairs with distance of 1, For any (e_i, e_j) , we can find a dominating (e_i, e_j) -trail in figure 1. Taking (e_0, e_4') as an example, we list the dominating edge trail sequence: $e_0 u_1 e_1 u_2 f_2 v_2 e_2' v_0 e_3' v_3 f_3 u_3 e_3 u_4 f_4 v_4 e_4'$. . Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 2.3 e_i is the edge on the outer C_5 of the Petersen graph, and e_j is the edge connecting the outer C_5 and the inner C_5. There are 10 different (e_i, e_j) edge pairs with distance of 1, which can be separated into four types: $(e_n, f_{n-1})(n=1,2,3,4)$, $(e_n, f_{n+2})(n = 0,1,2)$, $(e_n, f_{n-3})(n = 3,4)$, and (e_0, f_4) . For any (e_i, e_j) we can find a dominating (e_i, e_j) -trail in figure 1. Taking (e_0, f_2) as an example, we list the dominating edge trail sequence: $e_0 u_0 f_0 v_0 e_3' v_3 e_1' v_1 e_4' v_4 f_4 u_4 e_3 u_3 e_2 u_2 f_2$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 2.4 Both e_i and e_j are edges connecting the outer C_5 and the inner C_5 in the Petersen graph. There are 5 different (e_i, e_j) edge pairs with distance of 1, which can be separated into four types: $(f_n, f_{n+2})(n = 0,1,2)$, $(f_n, f_{n-3})(n = 3,4)$, $(f_n, f_{n+1})(n = 0,1,2,3)$, (f_0, f_4) . For any (e_i, e_j) , we can find a dominating (e_i, e_j) -trail in figure 1. Taking (f_0, f_4) as an example, we list the dominating edge trail sequence: $f_0 u_0 e_0 u_1 e_1 u_2 e_2 u_3 f_3 v_3 e_3' v_0 e_0' v_2 e_2' v_4 f_4$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 2.5 e_i is the edge connecting the outer C_5 and the inner C_5 in the Petersen graph, and e_j is the edge on the inner C_5. There are 10 different (e_i, e_j) edge pairs with distance of 1, which can be separated into four types: $(f_n, e'_{n+2})(n = 0,1,2)$, $(f_n, e'_{n+1})(n = 0,1,2,3)$, $(f_n, e'_{n-3})(n = 3,4)$, and (f_4, e_0) . For any (e_i, e_j), we can find a dominating (e_i, e_j)-trail in figure 1. Taking (f_0, e'_2) as an example, we list the dominating edge trail sequence: $f_0u_0e_4u_4f_4v_4e'_4v_1e'_1v_3f_3u_3e_2u_2f_2v_2e'_2$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 2.6 Both e_i and e_j are edges on the inner C_5 of the Petersen graph. There are 5 different (e_i, e_j) edge pairs with distance of 1, which can be separated into two types: $(e'_n, e'_{n+1})(n = 0,1,2,3)$, and (e'_0, e'_4) . For any (e_i, e_j) , we can find a dominating (e_i, e_j)-trail in figure 1. Taking (e'_0, e'_4) as an example, we list the dominating edge trail sequence: $e'_0v_2f_2u_2e_1u_1e_0u_0e_4u_4e_3u_3f_3v_3e'_1v_1e'_4$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Case 3. $dist(e_i, e_j) = 2$ in *G*, Since the *diam(G)* = 2 of the Petersen graph, so we can only provide 2 different cases in this condition.

Subcase 3.1 e_i is the edge on the outer C_5 of the Petersen graph, and e_j is the edge on the inner C_5. There are only 5 different (e_i, e_j) edge pairs with distance of 2, which can be separated into two types: $(e_n, e'_{n+2})(n = 0,1,2)$ and $(e_n, e'_{n-3})(n = 3,4)$. For any (e_i, e_j), we can find a dominating (e_i, e_j)-trail in figure 1. Taking (e_0, e'_2) as an example, the dominating edge trail sequence is: $e_0u_0f_0v_0e'_3v_3f_3u_3e_2u_2e_1u_1f_1v_1e'_4v_4e'_2$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

Subcase 3.2 e_i is the edge on the outer C_5 of the Petersen graph, and e_j is the edge connecting the outer C_5 and the inner C_5. There are only 5 different (e_i, e_j) edge pairs with distance of 2, which can be separated into two types: $(e_n, f_{n+3})(n = 0,1)$ and $(e_n, f_{n-2})(n = 2,3,4)$. For any (e_i, e_j), we can find a dominating (e_i, e_j)-trail in figure 1. Taking (e_0, f_3) as an example, the dominating edge trail sequence is: $e_0u_0f_0e'_0v_2f_2u_2e_1u_1f_1v_1e'_4v_4f_4u_4e_3u_3f_3$. Similarly, we can determine such dominating edge trail sequences for the remaining edge pairs.

According to theorem 2, we can sure that *L(G)* of Petersen graph is Hamilton-connected, that is *hc* (*G*)=1.

Lemma 19 [1] Whitney Theorem: There exists a relationship between $\kappa(G)$ and $\kappa(L(G)$ of *G*: $\kappa(L(G) \geq 2\kappa(G) - 2$.

Since multiedges are not permitted to appear in this passage, we didn't consider the graph obtained by replacing every vertex of Petersen graph with a 2-cycle. Since C_5 contains C_3, and C_3 is also K_3 , the graphs obtained by replacing each vertices of the Petersen graph with *n*-cycle (3 ≤ *n* ≤ 6) can be discussed in two cases:

Proof of Theorem 17. Let *G* be the graph which is obtained by replacing each vertex of the Petersen graph with a 3-cycle. It can be known that $\kappa(G) = 3$. According to theorem 5, let $\kappa^3(G) = \min\{m \mid L^m(G) \text{ is } 3-connected\}$ for this newly constructed graph. When *m*=0, we have $\kappa^3(G) = m = 0$, which satisfied the conditions of this theorem. Consequently, we can obtain that 0 ≤ *hc*(*G*) ≤ 2 for graph *G*. Then, by Theorem 15(II), the Hamiltonian indices *h(G)*=2. Combining this with the previous result, we can further get 2 ≤ *hc* (G) ≤ 2, that is, *hc (G)*=2(see figure 2).

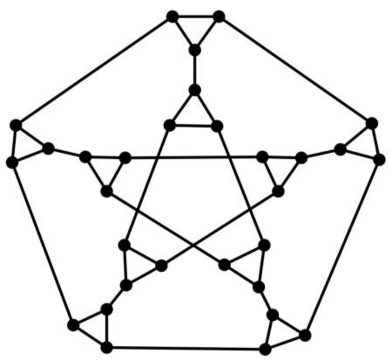

Figure 2: the graph obtained by replacing every vertex with a 3-cycle of Petersen graph

Case 2 Let *G* be the graph which is obtained by replacing each vertex of the Petersen graph with a *n*-cycle ($4 \le n \le 6$), it is obviously that $\kappa(G)=2$. We can observe that $\kappa(L(G)=3$. According to theorem 5, we have $\kappa^3(G)=1$, then $1 \le hc(G) \le 3$. The Hamiltonian indices $h(G) = 2$ by theorem 15(II). Therefore, we can further conclude that $2 \le hc(G) \le 3$. Let $G^{'}=L(G)$, since any three edges incident to a degree-3 vertex in *G* form a claw structure $K_{1,3}$, thus every degree-3 vertex in $G^{'}$ lies on a triangle, and $\kappa(G^{'})=3$. According to Lemma 19, $\kappa(L(G^{'})) \ge 2\kappa(G^{'})-2$, i.e., $\kappa(L(G^{'}) \ge 4$. By theorem 13, $L^2(G) = L(G^{'})$ is Hamilton-connected. Hence, $hc(G) = 2$.

Proof of Theorem 18. Let G be the graph which is obtained by replacing each vertex of the Petersen graph with an *n*-cycle ($n \ge 7$), by theorem 15(II), the Hamiltonian indices $h(G) = 2$ for this graph. Therefore, we can further conclude that $hc(G) \ge 2$.

4. Concluding Remarks

Determining the Hamilton-connected index *hc(G)* of a graph is *NP*-hard, there are few results on it. In this paper, we determine that the Hamilton-connected indices of the Petersen graph is 1, and the Hamilton-connected indices of graphs obtained by replacing each vertex of the Petersen graph with an *n*-cycle ($3 \le n \le 6$) is 2. When $n \ge 7$, there is no effective method to determine the Hamilton-connected index, and thus this problem remains a topic for future research.

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